VIU VANCOUVER ISLAND UNIVERSITY
EXPLORE. DISCOVER. EXCEL.

ALGEBRA REVIEW

Before taking Math 045/046, you should be familiar with all the topics included here. Read the explanations carefully and look over the examples provided.

Check your answers in the answer key at the end.

Do not use a calculator, unless you are instructed to.

ANSWER KEY AT THE END

CONTENTS

SECTION 1: SIGNED NUMBERS

In the REAL NUMBER SYSTEM, numbers can be either **positive or negative.**

Positive 5 can be written as: +5, or (+5), or just 5.

Negative 5 can be written as: –5, or (–5).

SECTION 2: ADDING and SUBTRACTING SIGNED NUMBERS

1. When adding numbers of the same sign, simply put the numbers together and carry the sign.

   $(+4)+(+2) = +6$ $\qquad$ $(-4)+(-2) = -6$

   $-4-2 = -6$ $\qquad$ $(-9)-5 = -14$

   Note: This reads –9 **combined with** –5

2. When adding numbers of different signs, take the difference between the two numbers; carry the sign of the number with the largest absolute value.

   For example: –9 + 2. The difference is 7. Take the sign of the 9. Answer = (–7)

   1) $4-2=2$
   2) $-2+4=2$
   3) $-7+3=-4$
   4) $12+(-3)=9$

   NOTE: These two examples are the same, just switched around!

EXERCISE ON SECTION 2: ADDING AND SUBTRACTING SIGNED NUMBERS

1) $-9+2$	2) $-3+(-5)$	3) $-5/8+1/4$
4) $-6+(-8)$	5) $0-10$	6) $-8-3$
7) $18-63$	8) $-49+(-4)$	9) $-2/3-3/4$
10) $(-5)+(12)-7$	11) $3.5-2.2+(-4.0)$	12) $15+(-2)-7+14-5+(-12)$

SECTION 3: MULTIPLYING AND DIVIDING WITH SIGNED NUMBERS

1. The rules for multiplying and dividing two numbers at a time are the same. They are as follows:

   I. When the signs are the same the answer is positive.

   II. When the signs are different the answer is negative.

1) $-6 \cdot (-5) = 30$

2) $6 \cdot (-5) = -30$

3) $(-4)(3) = -12$

4) $-\frac{14}{21} = -\frac{2}{3}$

5) $-36 \div -6 = 6$

6) $7 \div (-2) = -3.5$

7) $\frac{-25}{-5} = 5$

8) $-\frac{3}{4} = -0.75$

REMOVING BRACKETS

When doing operations with more than two signed numbers, remove brackets first before doing any operations. Then, simply take two numbers at a time.

To remove brackets: if a positive sign is outside the brackets simply remove the brackets.

e.g. +(7) becomes 7 *and* +(–7) becomes –7

if a negative sign is outside the brackets, **change the sign of the number in the brackets.**

e.g. – (+5) becomes –5 *and* – (–5) becomes 5

1) $+(-5) - (-7) = -5 + 7$
$= \mathbf{2}$

2) $-(-8) - (7) + (-2) = 8 - 7 - 2$
$= 1 - 2$
$= \mathbf{-1}$

EXERCISE ON SECTION 3: OPERATIONS WITH SIGNED NUMBERS

1) $-14 \cdot 7$ 2) $-9 \cdot -2$ 3) $-45 \div 9$

4) -6.3×2.5 5) $11 \div -2$ 6) $-72 \div -8$

7) $18-(-15)-(-5)$ 8) $-44+(\frac{3}{8})-(6)$ 9) $-(-8)+(-7)-(-\frac{1}{2})$

10) $(-14) \cdot (-3) \cdot (6)$ 11) $-(-2)+8-7-(-24)+(-7)$ 12) $84+(-99)+44-(-18)-43$

SECTION 4: EXPONENTS

Before looking at ORDER OF OPERATIONS, it is necessary to understand how exponents function. Consider the following:

5^3 — 3 is the exponent; 5 is the base

This is read 5 to the third power, and it means: $5 \times 5 \times 5 = 125$

As an introduction to algebra, the same is done with letters: $Y \cdot Y \cdot Y = Y^3$

Special case : Anything to the zero power is equal to 1.

a) $(3)^0 = 1$ b) $(-3)^0 = 1$ c) $x^0 = 1$ d) $(-x)^0 = 1$

Examples:

1) $(-3)^3 = -27$ 2) $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ 3) $(5x)^3 = 125x^3$

Note ⟶ 4) $\mathbf{-2^2 = -(2 \times 2) = -4}$ but $\mathbf{(-2)^2 = -2 \times -2 = +4}$

EXERCISE ON SECTION 4: EXPONENTS

Evaluate or write without brackets.

1) 4^2 2) 1^5 3) $(-1)^6$ 4) $(-2)^4$ 5) $(x)^3$ 6) $(6x)^2$

7) $\left(\frac{1}{3}\right)^2$ 8) $\frac{5^2}{8}$ 9) $(4y)^3$ 10) $(-2xy)^3$ 11) $-(-3z)^2$

SECTION 5: ORDER OF OPERATIONS

When different operations are combined in one problem, they must be done in a certain order. The acronym for remembering the order of operations is **BEMA.**

B - Brackets. All operations inside brackets must be done first.
For example: $2(3 + 5) = 2(\mathbf{8}) = 16$.

E - Exponents. An exponent indicates how many times to multiply a number by itself. Any exponents must be carried out next.
For example: $2(3^2 + 5) = 2(\mathbf{9} + 5) = 2(\mathbf{14}) = 28$

M - Multiplication and division must be carried out before adding and subtracting.
For example: $2 \cdot 3 + 4 \div 2 = \mathbf{6} + \mathbf{2} = 8$

A - Adding and subtracting are done last, as in the previous example.
For example: $6 + 3 \cdot 5 - 1 = 6 + \mathbf{15} - 1 = \mathbf{21} - 1 = 20$

Fill in the missing steps to this example:

$$\frac{-2(10 - 6^2)}{3^2 \cdot 3^2 - 1} = \frac{-2(10 - \quad)}{\bullet \quad - 1}$$

$$= \frac{-2(- \quad 6 \;)}{-1}$$

$$= \frac{13}{20}$$

EXERCISE ON SECTION 5: ORDER OF OPERATIONS

1) $(8 - 2)(3 - 9)$

2) $(-1)^4 + 2^3 - 10$

3) $8 - (2 \cdot 3 - 9)$

4) $-7(3^4) + 18$

5) $6\,[\,9 - (3 - 4)]$

6) $4 \cdot 5 - 2 \cdot 6 + 4$

7) $9 \div (-3) + 16 \cdot (-2) - 1$

8) $\dfrac{5^2 - 4^3}{9^2 - 2^2}$

9) $\dfrac{(3 - 5)^2 - (7 - 13)}{(12 - 9)^2 + (11 - 14)}$

10) $[-12(-3) - 2^3] - (-9)(-10)$

11) $-2(16) - [\,2(-8) - 5^3\,]$

12) $3(-4.5) + (2^2 - 3 \cdot 4^2)$

SECTION 6: THE THREE RULES OF EXPONENTS

Rule 1: WHEN MULTIPLYING WITH THE SAME BASE, ADD THE POWERS.

1) $x^2 \cdot x^3 = x^{2+3} = x^5$

2) $4^2 \cdot 4^3 = 4^{2+3} = 4^5$

3) $a \cdot a^4 \cdot a = a^{1+4+1} = a^6$

4) $-y \cdot y^3 = -y^{1+3} = -y^4$

NOTICE, EVEN THOUGH THE EXPONENT OF 1 IS NOT WRITTEN, IT IS STILL THERE AND MUST BE ADDED.

When multiplying two terms such as $2x^2$ and $5x^4$, multiply the numbers and add the powers.

5) $(2x^2)(5x^4) = 10x^{2+4} = 10x^6$

6) $(-3ab^2)(-4a^2b) = 12a^3b^3$

Rule 2: WHEN DIVIDING WITH THE SAME BASE, SUBTRACT THE EXPONENTS

1) $\frac{2^5}{2^3} = 2^{5-3} = 2^2$ or 4

2) $\frac{a^8}{a} = a^{8-1} = a^7$

3) $\frac{b^4}{b^4} = b^0$ or 1

4) $\frac{-20x^5a^2}{5x^2a} = -4x^3a$

Rule 3: WHEN A POWER IS OUTSIDE BRACKETS, THE POWERS ARE MULTIPLIED

1.) $(c^2)^3 = c^{2 \cdot 3} = c^6$

2.) $(3x^3)^2 = 9x^6$

3.) $(5x^2y^4)^2 = 25x^4y^8$

EXERCISE ON SECTION 6: USING EXPONENTS

1) $2^2 \cdot 2^5$

2) $x^2 \cdot x \cdot x^3$

3) $2a \cdot 5a^3$

4) $3z^2 \cdot -4z^5$

5) $\frac{z^{12}}{z^3}$

6) $\frac{-6a^7z^2}{3a^3z}$

7) $(-7ab^3)(-3b)$

8) $a \cdot a^3 \cdot a^2$

9) $(m^3)^7$

10) $(3x^5)^3$

11) $(-3xy)(-6x^3)$

12) $\frac{-25xy^5}{-10xy}$

13) $y^3 \cdot (5x^4y)^2$

14) $\frac{x \cdot (2x^4y^2)^3}{x^2 \cdot 3xy^3}$

15) $-(-3a^5b)^3 + (-2a^{15}b^3)$

SECTION 7: SOLVING EQUATIONS - *ADDITION AND SUBTRACTION*

An equation is solved when the unknown letter is isolated on one side of the equal sign. When isolating x, the equation must be kept balanced. To maintain balance, you must always do the same thing to both sides of the equation.

For example: Solve for x: $x + 6 = 32 \longrightarrow$ to isolate x, we must subtract 6 from the left, and thus from the right.

$$x + 6 - \mathbf{6} = 32 - \mathbf{6}$$
$$x = 26$$

Another example:

$$45 = -12 - x$$
$$45 + 12 = -x$$
$$57 = -x \quad \text{[Note: we want to solve for +x, not –x]}$$
$$-57 = x$$

EXERCISE ON SECTION 7: EQUATIONS WITH ADDITION AND SUBTRACTION

Solve for the missing letter.

1) $z - 3 = 25$

2) $a + 6.5 = 0.009$

3) $-34 = -6 - y$

4) $\frac{-3}{20} = y - 6$

5) $(-x) + \frac{4}{7} = -\frac{1}{3}$

6) $-9.65 = 0.8 - x$

7) $436 = a - 58$

8) $-9.6 - x + 3.4 = \frac{1}{2} - 3$

9) $-(6 + x) = \frac{2}{3} + (-7)$

10) $\frac{3}{4} - \frac{7}{8} = x + 0.9$

SECTION 8: SOLVING EQUATIONS - *MULTIPLICATION AND DIVISION*

These types of equations always have a number greater than 1 in front of the letter, or unknown. In the previous type of equation all you had to do was get your letter on one side of the equals sign and all the numbers on the other, and you were done! This is always your first step! Now, if your coefficient is greater than 1, you must divide both sides of the equal sign by the number in front of the letter. Here are three examples of how to tell what the coefficient is:

e.g. $-3x$ — coefficient = (-3)

e.g. $\frac{3x}{5}$ — coefficient = $\left(\frac{3}{5}\right)$

e.g. $\frac{x}{4}$ — coefficient = $\left(\frac{1}{4}\right)$

In this section, we will just concentrate on the second step for solving equations. In the next section, we will combine both steps.

1) $6z = -9$

$\frac{6z}{6} = \frac{-9}{6}$

$z = -\left(\frac{3}{2}\right)$

2) $\frac{1}{3}x = 5$

$\frac{3}{1} \cdot \frac{1}{3}x = 5 \cdot \frac{3}{1}$

$x = 15$

3) $\frac{x}{5} = \frac{4}{5}$

$\frac{5}{1} \cdot \frac{x}{5} = \frac{4}{5} \cdot \frac{5}{1}$

$x = 4$

EXERCISE ON SECTION 8: EQUATIONS WITH MULTIPLICATION AND DIVISION

1) $2x = -16$

2) $-4y = -35$

3) $-86 = 5z$

4) $\frac{x}{2} = \frac{3}{5}$

5) $2.56b = -1.28$

6) $\frac{2}{3}x = 5$

7) $-\frac{1}{3}t = 7$

8) $50 = -x$

9) $-\frac{2r}{3} = -\frac{27}{4}$

10) $\frac{x}{-5} = (-12.06)$

SECTION 9: SOLVING EQUATIONS – *BOTH METHODS TOGETHER*

Before we look at the combination of both methods, we must first review adding and subtracting **like terms.** When adding terms that have the **same letter and same exponent, add the coefficients and carry the letter.**

e.g. $2x + 3x = 5x$ e.g. $6x - x = 5x$ e.g. $-4y - 3y = -7y$

TWO STEPS TO SOLVE EQUATIONS:

1.) COLLECT ALL NUMBERS ON ONE SIDE OF THE EQUAL SIGN, AND COLLECT ALL LETTERS ON THE OTHER SIDE.
2.) SIMPLIFY AND DIVIDE BY THE NUMBER IN FRONT OF THE LETTER.

1) Solve:

$$\begin{aligned} 3x + 4 &= 13 \\ 3x &= 13 - 4 \\ 3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} \\ x &= 3 \end{aligned}$$

2)

$$\begin{aligned} 2x - 2 &= -3x + 3 \\ 2x - 2 + 2 &= -3X + 3 + 2 \\ 2x &= -3x + 5 \\ 2x + 3x &= -3x + 3x + 5 \\ 5x &= 5 \\ \frac{5x}{5} &= \frac{5}{5} \\ x &= 1 \end{aligned}$$

EXERCISE ON SECTION 9: EQUATIONS WITH BOTH PRINCIPLES.

1) $5x + 6 = 31$ 2) $8x + 4 = 68$ 3) $-5y - 7 = 108$

4) $-91 = 9t + 8$ 5) $5x + 7x = 72$ 6) $-4y - 8y = 48$

7) $x + \frac{1}{3}x = 8$ 8) $5x + 3 = 2x + 15$ 9) $2x - 1 = 4 + x$

10) $4 + 3x - 6 = 3x + 2 - x$ 11) $5y - 7 + y = 7y + 21 - 5y$

ANSWERS

SECTION 2

1) -7 2) -8 3) $-\frac{3}{8}$ 4) -14 5) -10

6) -11 7) -45 8) -53 9) $-1\frac{5}{12}$ 10) 0

11) -2.7 12) 3

SECTION 3

1) -98 2) 18 3) -5 4) -15.75 5) -5.5

6) 9 7) 38 8) $-49\frac{5}{8}$ 9) $1\frac{1}{2}$ 10) 252

11) 20 12) 4

SECTION 4

1) 16 2) 1 3) 1 4) 16 5) x^3

6) $36x^2$ 7) $\frac{1}{9}$ 8) $3\frac{1}{8}$ 9) $64y^3$ 10) $-8x^3y^3$

11) $-9z^2$

SECTION 5

1) -36 2) -1 3) 11 4) -549 5) 60 6) 12

7) -36 8) $-\frac{39}{77}$ 9) $\frac{5}{3}$ or $1\frac{2}{3}$ 10) -62 11) 109 12) -57.5

SECTION 6

1) 2^7 2) x^6 3) $10a^4$ 4) $-12z^7$ 5) z^9 6) $-2a^4z$

7) $21ab^4$ 8) a^6 9) m^{21} 10) $27x^{15}$ 11) $18x^4y$ 12) $2.5y^4$

13) $25x^8y^5$ 14) $\frac{8}{3}x^{10}y^3$ 15) $25a^{15}b^3$

SECTION 7

1) 28 2) -6.491 3) 28 4) $5\frac{17}{20}$ 5) $\frac{19}{21}$ 6) 10.45

7) 494 8) -3.7 9) $\frac{1}{3}$ 10) –1.025

SECTION 8

1) –8 2) 8.75 3) –17.2 4) $1\frac{1}{5}$ 5) –0.5 6) $7\frac{1}{2}$

7) –21 8) –50 9) $10\frac{1}{8}$ 10) 60.3

SECTION 9

1) 5 2) 8 3) –23 4) –11 5) 6 6) – 4

7) 6 8) 4 9) 5 10) 4 11) 7